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ON ACOUSTIC BOUNDARY-CONTACT PROBLEMS FOR A VERTICALLY STRATIFIED MEDIUM BOUNDED FROM ABOVE BY A PLATE WITH CONCENTRATED INHOMOGENEITIES*

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An approach is proposed to the description of diffraction fields of acoustic waves in a vertically stratified medium covered from above by an elastic plate. There is a finite number of point of parallel linear inhomogeneities (cracks, concentrated masses and moments of inertia, etc.) on the plate. Analogous investigations were performed earlier for a homogeneous medium by using integral transformation techniques. The procedure for the numerical solution of one-dimensional differential equations should also be relied upon for the media under consideration.

The boundary-value problem for the Helmholtz equation for whose formulation conditions must be given at individual points or on individual lines of the boundary (boundary contact conditions or BCC)** (**Belinskii B.P., On a Regularization Method in Diffraction Problems by Reinforce Plates. Doctoral Dissertation, Leningrad State University, 1986.) in addition to the boundary conditions, is called a boundary-contact problem of acoustics (BCP). An integral transformation technique was developed (/1/***(***See also Kouzov D.P., Boundary-Contact Problems of Acoustics (Plate-Fluid System). Doctoral Dissertation Institute of Acoustics, Moscow, 1986), etc.) that enabled the solution of a broad class of BCP to be constructed in quadratures. The fact that the boundary condition can be written (in terms of generalized functions) on the whole plate in the presence of just point (linear) inhomogeneities on the plate, but a linear combination of delta functions and their derivatives concentrated on the inhomogeneities occurs in the right side, is used here. By using the Fourier integral transformation method, it is possible to arrive at a field representation that contains coefficients of the above-mentioned linear combination (boundary-contact constants). To determine them one need only rely on a specific BCC on each inhomogeneity that fixes the mechanical mode thereon. Finally, a finite system of lienar algebra equations (BCS) is obtained for the boundary-contact constants. The BCP for a homogeneous medium in the presence of cracks or stiffness ribs on the plate /1-4/ were solved explicitly by the means described and were analysed physically in detail.

From the viewpoint of applications, the transfer of the scheme described above to the case of an inhomogeneous fluid is of interest. This is done in this paper for a medium stratified according to depth by an example of cylindrical wave diffraction by a plate with cracks (the plane problem). A difference is noted in the energy transport mechanism as compared with a homogeneous medium. An optical theorem that is a convenient checking identity for the field computation is formulated for this model. A single-valued solvability of the BCS and a uniqueness theorem for solving it when there is no absorption in the medium and the plate are established.

1. Let an acoustic half-space $(z > 0, -\infty < x < \infty)$ be covered from above by a thin flexibly-vibrating plate $(z = 0, -\infty < x < \infty)$ weakened by a system of cracks $(z = 0, x = a_n, n = 1, \ldots, N)$. The acoustic medium consists of a stratified layer of depth H under which there is a homogeneous bottom $(z > H)$. The pressure in the medium satisfies the Helmholtz equation

$$(\Delta + k^2(z))\, g(x, z) = \delta(x - x_0)\, \delta(z - z_0) \tag{1.1}$$

($\delta(x)$ is the delta function), where it is assumed that $z_0 < H$. The equation becomes homogeneous on the bottom, hence $k(z) = k_1$. The continuity conditions

$$[g(x, H)] = 0, \quad g'(x, H - 0) = \varkappa^{-1} g'(x, H + 0), \quad \varkappa = \rho_1/\rho \tag{1.2}$$

are satisfied on the surface of the bottom. Here and henceforth $[f(t_0)]$ is the jump in the function $f(t)$ at the point $t = t_0$, the prime denotes a derivative with respect to the argument z, and ρ and ρ_1 are the layer and bottom densities. The boundary condition on the plate has the form /1/

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$$\left(\frac{\partial^4}{\partial x^4} - k_0^4\right) g'(x, 0) + \nu g(x, 0) = \sum_{n=1}^{N} \sum_{j=0}^{3} c_{nj} \frac{d^j \delta(x - a_n)}{dx^j} \tag{1.3}$$

$$k_0^4 = \mu\omega^2/D, \quad \nu = \rho\omega^2/D$$

Here c_{nj} are the boundary-contact constants, μ is the plate mass per unit length, D is the cylindrical stiffness, and ω is the frequency. The solution is understood in the sense of the limit absorption.

The boundary-value problem (1.1)-(1.3) is supplemented by some kinds of BCC at the inhomogeneities. We will present just two examples.

Crack: the BCC are, when there are no transverse forces and no bending moment at the crack edges $(x = a_n \pm 0)$ /1/

$$\lim_{x \to a_n \pm 0} \frac{\partial^3 g'(x, 0)}{\partial x^3} = \lim_{x \to a_n \pm 0} \frac{\partial^2 g'(x, 0)}{\partial x^2} = 0, \quad 1 \leqslant n \leqslant N \tag{1.4}$$

Stiffness rib or hummock: the BCC are derived from the equations of motion of a body attached rigidly to the plate at the point $x = a_n$ /3/

$$D\,[\partial^3 g'(a_n, 0)/\partial x^3] = M\omega^2 g'(a_n, 0) \tag{1.5}$$

$$D\,[\partial^2 g'(a_n, 0)/\partial x^2] = -J\omega^2 \partial g'(a_n, 0)/\partial x$$

Here M, J are the rib mass and the moment of inertia with respect to the axis passing through the point of fastening to the plate. It is assumed here that kinematic continuity conditions are satisfied

$$[\partial g'(a_n, 0)/\partial x] = [g'(a_n, 0)] = 0 \tag{1.6}$$

2. To be specific, the BCC case (1.4) is examined below. The solution is constructed by analogy with the case of a homogeneous half-space $(k(z) = k_1, \rho = \rho_1)$ by using the Fourier transform. The field g_0 that occurs when there are no cracks is extracted (to be specific we assume $z < z_0$)

$$g = g_0 + g_1 \tag{2.1}$$

$$g_0 = \frac{1}{2\pi i} \int e^{i\lambda(x - x_0)} \frac{\psi_0(z, \lambda)\,\psi_1(z_0, \lambda)}{W(\psi_0, \psi_1)}\, d\lambda$$

Here and henceforth the absence of limits of integration means that it is performed over the whole axis, ψ_0 and ψ_1 are solutions of the "depth" equation

$$\psi'' + (k^2(z) - \lambda^2)\,\psi = 0 \tag{2.2}$$

satisfying the boundary conditions

$$(\lambda^4 - k_0^4)\,\psi'(0, \lambda) + \nu\psi(0, \lambda) = 0 \tag{2.3}$$

$$\psi'(H, \lambda) + \varkappa^{-1} \sqrt{\lambda^2 - k_1^2}\,\psi(H, \lambda) = 0$$

respectively, and $W(\psi_0, \psi_1)$ is their Wronskian. The first of conditions (2.3) occurs because of the Fourier transformation of the homogeneous boundary condition (1.3), and the second because of the continuity conditions (1.2), taking the explicit solvability of (2.2) of the bottom into account

$$\psi_1(z, \lambda) = \psi_1(H, \lambda) \exp\{-\sqrt{\lambda^2 - k_1^2}\,(z - H)\}, \quad z > H \tag{2.4}$$

The scattered field g_1 will be sought in the form of a Fourier integral with unknown density $p(\lambda)$

$$g_1 = \frac{1}{2\pi i} \int e^{i\lambda x} p(\lambda) \frac{\psi_1(z, \lambda)}{l(\lambda)}\, d\lambda \tag{2.5}$$

$$l(\lambda) = (\lambda^4 - k_0^4)\,\psi_1'(0, \lambda) + \nu\psi_1(0, \lambda)$$

(the symbol of the boundary operator (1.3) is introdued here). The representation (2.5) is a natural extension of the field representation in the case of a homogeneous half-space when the function $\psi_1(z, \lambda)$ is determined from (2.4) with $H = 0$ /1-3/ (see also the papers mentioned in footnotes 1 and 2). The field g_1 satisfies the homogeneous Helmholtz equation and conditions (1.2). The boundary condition (1.3) yields

$$p(\lambda) = c_{nj} e^{-i\lambda a_n} (i\lambda)^j \tag{2.6}$$

Here and henceforth, summation over repeated subscripts is understood: between the limits from 1 to N for n and from 0 to 3 for j.

It is important to note that the scattered field representations (2.5) and (2.6) can be used equally in the solution of BCP with any point inhomogeneities on the boundary.

3. Let us consider the BCC (1.4) for the case of cracks at all points $x = a_m$. We find $(s = 2, 3)$

$$\lim_{x \to a_m \pm 0} \frac{\partial^s g'(x, 0)}{\partial x^s} = \lim_{x \to a_m \pm 0} \frac{\partial^s g_0'(x, 0)}{\partial x^s} + \frac{1}{2\pi i} c_{nj} I^{\pm}_{mns+j}$$

The boundary-contact integrals derived here

$$I^{\pm}_{mnj} = \int e^{i\lambda(a_m - a_n \pm 0)} \frac{\psi_1'(0, \lambda)}{l(\lambda)} (i\lambda)^j \, d\lambda \tag{3.1}$$

diverge for $j > 2$. The procedure for their regularization that occurs systematically for the solution of BCP for a homogeneous medium /1-3/ is described in Sect.4. The derivatives of the source with respect to the field are expressed by using the elementary identity $\nu W(\psi_0, \psi_1) = \psi_0'(0, \lambda)\, l(\lambda)$ by the convergent integrals

$$\frac{\partial^s g_0'(x_m, 0)}{\partial x^s} = \frac{J_{ms}}{2\pi i}, \quad J_{ms} = \nu \int e^{i\lambda(a_m - x_0)} \frac{\psi_1'(z_0, \lambda)}{l(\lambda)} (i\lambda)^s \, d\lambda$$

The convergence here is due to the exponential decrease of the function $\psi_1(z_0, \lambda)$, as $\lambda \to \pm\infty$ as is easily confirmed by the WKB method*. (*Buldyrev V.S. and Buslayev V.S., Sound Propagation in the Ocean, Preprint 45(417). USSR Academy of Sciences, Institute of Radio Engineering and Electronics, Moscow, 1984.).

The BCC (1.4) result in a BCS of $4N$ linear algebraic equations for $4N$ coefficients c_{nj}

$$I^{\pm}_{mnj+3} c_{nj} = -J_{m3}, \quad I^{\pm}_{mnj+2} c_{nj} = -J_{m2} \tag{3.2}$$

The solvability of this system is discussed in Sect.6.

The complete solution of the original BCP is given by (2.1), (2.5), (2.6) and (3.2).

4. We will now consider the procedure for regularizing the integrals (3.1). We understand $g'(x, 0)$ in (1.4)-(1.6) to be the limit as $z \to 0$. Then the integrals $I^{\pm}_{mnj}$ $(n \neq m)$ result in convergent deformations of the contour of integration in the upper half-plane λ for $a_m > a_n$ or the lower half-plane for $a_m < a_n$ prior to passage to the limit in z. After this passage the convergence is due to the decrease in the exponential function $\exp(i\lambda(a_m - a_n))$.

The integrals $I^{\pm}_{mmj}$ are independent of m. We set $I^{\pm}_{mmj} = I_j^{\pm}$. It is clear that $I_j^- = (-1)^j I_j^+$. We limit ourselves to regularization of I_j^+ and for brevity we omit the superscript "plus", below. In the case of a homogeneous medium the regularization of analogous integrals is performed in two steps. First the contour of integration is deformed into a loop enclosing the slit for $(\lambda^2 - k_1^2)^{1/2}$ in the upper λ half-plane. Then the integral in the whole loop is replaced by an integral over one of its edges from the jump in the integrand on the slit. Consequently, convergent integrals occur that later reduce to sums of residues at poles of the integrand lying in the upper half-planes of a two-sheeted Riemann surface /1/. Because of the conservation of the analytic properties of the integrand during passage to a stratified medium, with the exception of poles associated with the presence of a wave channel $(0 < z < H)$ that occur in addition (poles of the integrand are discussed in Sect.5), the procedure described here remains applicable. We will just present the result

$$I_{2p+1} = {}^1/_2 \pi i \nu \sum_l (i\lambda_l)^{2p+1} R_l \tag{4.1}$$

$$I_{2p} = {}^1/_2 \nu \sum_l (i\lambda_l)^{2p} R_l \left(\pi i + 2 \ln\left(\left(\lambda_l + \sqrt{\lambda_l^2 - k_1^2}\right)/k_1\right)\right) \tag{4.2}$$

The quantities R_l are determined by the eigenfunctions ψ_l and eigenvalues λ_l of the spectral problem (2.2) and (2.3)

$$R_l = (\lambda_l^4 - k_0^4)^{-2} \frac{\psi_l^2(0)}{\psi_l(H)} \left\{ \frac{\partial \psi_0'(H, \lambda_l)}{\partial \lambda} + \frac{1}{\varkappa} \sqrt{\lambda_l^2 - k_1^2} \times \frac{\partial \psi_0(H, \lambda_l)}{\partial \lambda} + \frac{\lambda_l \psi_l(H)/\varkappa}{\sqrt{\lambda_l^2 - k_1^2}} \right\}^{-1} \tag{4.3}$$

In the case of a homogeneous half-space, the integrals I_3 and I_5 can be evaluated analytically /5/

$$I_3 = \pi, \; I_5 = 0 \tag{4.4}$$

To prove the validity of the result (4.4) in the case under consideration, we extract the growing parts from the integrands of I_3 and I_5 and separate the integrals into two components

$$I_j = \int e^{+i\lambda 0} \frac{(i\lambda)^j}{\lambda^4 - k_0^4}\, d\lambda - \nu \int \frac{\psi_1(0, \lambda)}{l(\lambda)} \frac{(i\lambda)^j}{\lambda^4 - k_0^4}\, d\lambda$$

The integrals in the second components converge for $j < 7$ and for odd j because of the oddness of the integrand. Evaluation of the first components by residues yields (1.4).

5. We examine the case of one crack $(N = 1)$ by setting $a_1 = 0$ (we omit the subscript $n = 1$ below). The BCS (3.2) dissociates into two independent BCS after symmetrization and antisymmetrization

$$\begin{Vmatrix} c_0 \\ c_2 \end{Vmatrix} = - \begin{Vmatrix} I_3 & I_5 \\ I_2 & I_4 \end{Vmatrix}^{-1} \begin{Vmatrix} 0 \\ J_2 \end{Vmatrix}, \quad \begin{Vmatrix} c_1 \\ c_3 \end{Vmatrix} = - \begin{Vmatrix} I_4 & I_6 \\ I_3 & I_5 \end{Vmatrix}^{-1} \begin{Vmatrix} J_3 \\ 0 \end{Vmatrix}$$

In view of (4.4) the solution of these BCS has the form $c_0 = c_1 = 0$, $c_2 = -J_2/I_4$ and $c_3 = -J_3/I_6$.

The exact analytic solution of the original BCP for the case of one crack has thereby been constructed.

We will make an asymptotic investigation of this solution as $kh \to 0$ (h is the plate thickness on which D, ν and k_0^4 depend) for the case of a Pekeris waveguide ($k(z) = k$, $0 < z < H$). We will investigate just the scattered field g_1. The solutions of the "depth" problem ψ_0 and ψ_1 can be expressed in terms of linearly independent solutions ψ_+ and ψ_- of Eq.(2.2)

$$\psi_\pm = \exp\left(\pm \sqrt{\lambda^2 - k^2}\, z\right) \tag{5.1}$$

Taking (5.1) into account we can rewrite the equation $l(\lambda) = 0$ (see the second formula in (2.5)) in the form

$$\sqrt{\lambda^2 - k^2} \cdot \operatorname{ctg}\left(\sqrt{\lambda^2 - k^2} \cdot H\right) = - \frac{(\lambda^4 - k_0^4)(\lambda^2 - k^2) - \nu \varkappa^{-1} \sqrt{\lambda^2 - k_1^2}}{\varkappa^{-1} (\lambda^4 - k_0^4) \sqrt{\lambda^2 - k_1^2} - \nu} \tag{5.2}$$

We introduce the parameter $\varepsilon = (kh)^{2/5} H/h$. Let $\varepsilon \ll 1$. Then by using elementary perturbation theory the asymptotic forms of the two series of roots of the dispersion equation ($\nu_0 = \nu (kh)^{-2}$) can be found

$$\lambda_l^{(1)} \sim \pm h^{-1} (kh)^{2/5} (\nu_0 \varkappa)^{1/5} e^{2\pi i l/5}, \; 0 \leqslant l \leqslant 4 \tag{5.3}$$

$$\lambda_l^{(2)} \sim \pm \left(\frac{1}{2H} \ln\left(\frac{\varkappa - 1}{\varkappa + 1} \right) + \frac{\pi i l}{H} \right), \quad |l| < \infty \tag{5.4}$$

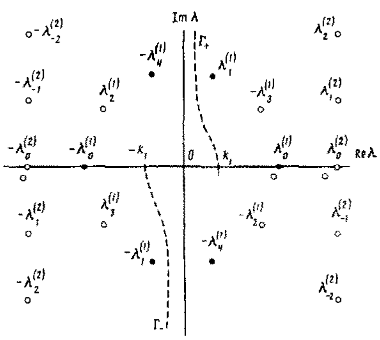
The roots $\lambda_l^{(1)}$ are at the vertices of a regular pentagon as $kh \to 0$, where $\pm\lambda_0^{(1)}$ corresponds to the surface wave process. The roots $\lambda_l^{(2)}$ are waveguide resonances (there are no real waveguide eigenvalues for $\varepsilon \ll 1$). The eigenvalues lying on the physical sheet of the Riemann surface are represented by dark points in the figure while the open circles are those lying on its non-physical sheet, $\Gamma_\pm$ (the dashed lines) are slits for $(\lambda^2 - k_1^2)^{1/2}$.

Let us evaluate the integrals I_4 and I_6 by residues according to (4.2). Elementary calculations show that as $kh \to 0$, $\varepsilon \ll 1$ only residues at the poles $\lambda = \lambda_l^{(1)}$ exert any influence on the highest terms of the asymptotic forms of these integrals (the poles $\lambda = \lambda_l^{(2)}$ are of the order $(kh)^{-\delta}$, $\delta \geqslant 0$, and the residues there are small compared with the residues at the poles $\lambda = \lambda_l^{(1)}$). Therefore, the highest terms of the asymptotic forms of the integrals in the representation for the scattered field g_1 agree with the highest terms of analogous integrals in the case of a homogeneous medium. Therefore, a thin water layer ($\varepsilon \ll 1$) is not felt by a scattered field at the highest order. In particular, the highest term of the asymptotic form of a field excited by a plane wave incident on a homogeneous bottom is identical with that presented in /1/.

The nature of the dependence of the layer parameters on the depth is obviously not essential here.

In order to see this we will examine the dispersion Eq.(5.2) whose left-hand side should be replaced by the logarithmic derivative of a certain solution (2.2). For $\lambda = 0\,(k)$ this derivative is of the order of one and, therefore, the asymptotic form $\lambda_l^{(1)}$ like the asymptotic form of the roots of the denominator of the right-hand side are conserved. The roots $\lambda_l^{(2)}$ are of the previous order and the residues therein are not felt by the highest terms of I_4 and I_6.

For $kH \gg 1$ there is a certain number L of real waveguide eigenvalues $\pm\lambda_l^{(2)}\ (0 \leqslant l < L)$ lying in the intervals $[k_1, k]$ and $[-k, -k_1]$ and four series of complex roots $\pm\lambda_l^{(2)}\ (|\,l\,| \geqslant L)$ departing from the points $\lambda = \pm k_1$ towards infinity. The asymptotic form of these roots of number l has the form of (5.4). The roots $\lambda_l^{(1)}$ are not described by the asymptotic form (5.3). It can merely be asserted that $\lambda_0^{(1)}$ turns out to be to the right of k as $kh \to 0$. A numerical analysis is necessary in this domain of parameters.



6. The solvability of the BCS (3.2) and, therefore, the theorem on the existence and uniqueness of the solution of the BCP (1.1)-(2.4) when the explicitly written solution (2.1), (2.5), (2.6) is taken into account, are proved in the same way as indicated in /6/.

We will refine the radiation principle by indicating the asymptotic form of the solution at infinity. The field is represented in the form of a sum of a surface (waveguide) process $g^\pm$ and a cylindrical wave g_c diverging at the bottom whose asymptotic form have the form

$$g^\pm = \sum_{l=0}^{L} \alpha_l^\pm e^{i\lambda_l|x|}\psi_l(z)\quad (1 + O(|\,x\,|^{-1/2})),\quad x \to \pm\infty$$

$$g_c = \sqrt{2\pi/k_1 r}\,\Phi(\varphi)\,e^{ik_1 r} + o\,(r^{-1/2}),\quad r = \sqrt{x^2 + (z - H)^2} \to \infty$$

where $\alpha_l^\pm$ are the amplitudes of the normal waves propagating in the waveguide from left to right from the domain of the plate occupied by the inhomogeneities, and $\Phi(\varphi)$ is the radiation pattern of a cylindrical wave.

By using the Green's first formula for the solution u of the homogeneous BCS in the domain $\Omega_{R,\delta}$ formed by the arc of the circle $S_R = \{x^2 + (z - H)^2 = R^2\}$ at the bottom, the segments $\{x = \pm R,\ 0 < z < H\}$, and a plate whose inhomogeneities are bypassed by the arcs $S_\delta^n = \{(x - a_n)^2 + z^2 = \delta^2,\ z > 0\}$ in the first stage of the proof, the absence of energy-carrying field components at infinity is established

$$\alpha_l^\pm = 0,\ l = 0, 1, \ldots, L;\quad \Phi(\varphi) = 0 \tag{6.1}$$

The property of orthogonality of the eigenfunctions of the spectral problem (2.2) and (2.3) is used to prove this assertion (δ_p^q is the Kronecker delta)

$$\int_0^H \psi_p(z)\,\bar\psi_q(z)\,dz + \frac{\lambda_p^2 + \lambda_q^2}{\nu}\,\psi_p'(0)\,\bar\psi_q'(0) + \varkappa^{-1}\left(\sqrt{\lambda_p^2 - k_1^2} + \sqrt{\lambda_q^2 - k_1^2}\right)^{-1}\psi_p(H)\,\bar\psi_q(H) = \delta_p^q \tag{6.2}$$

Two cases are considered further. 1. The BCS determinant is non-zero. Then a Green's function g exists. Applying the Green's second formula to it and the assumed solution of the homogeneous problem u, we establish the uniqueness of the solution (the solution of the homogeneous BCP equals zero identically). 2. If the BCS determinant equals zero, we construct a field u according to the existing non-zero solution c_{nj} of the homogeneous BCS. Evaluating the radiation pattern of this solution

$$\Phi(\varphi) = e^{-i\pi/4}\frac{k_1 \sin\varphi}{l\,(k_1\cos\varphi)}\,c_{nj}e^{ik_1 a_n\cos\varphi}(k_1\cos\varphi)^j$$

we arrive at a contradiction to (6.1) because not all the constants c_{nj} equal zero.

Therefore, a solution of the BCS, and therefore, of the BCP exists and is unique.

7. Let a plane wave be incident at an angle φ_0 from the bottom. Calculations analogous to those performed in the papers mentioned in footnotes 1 and 2 result in the following expression for the effective scattering cross-section of this wave on an infinite plate with arbitrary local non-absorbing inhomogeneities, covering the sound channel:

$$\sigma = \frac{2\pi}{k_1}\int_0^\pi |\,\Phi(\varphi, \varphi_0)\,|^2\,d\varphi + \frac{1}{k_1}\sum_{l=0}^{L}(|\,\alpha_l^+\,|^2 + |\,\alpha_l^-\,|^2)\,\lambda_l \tag{7.1}$$

The property of normalization of the eigenfunctions of the "depth" problem (6.2) was used in deriving (7.1). On the other hand, the scattering diameter is expressed in terms of the reflection coefficient

$$R(\varphi) = R^-(\varphi)/R^+(\varphi),\ R^{\pm} = \psi_0(H, k_1\cos\varphi)\sin\varphi \pm \varkappa^{-1}\psi_0'(H, k_1\cos\varphi)$$

and the value of the pattern Φ in the direction of the reflected wave

$$\sigma = -4\pi k_1^{-1}\,\mathrm{Re}\,(\bar{R}(\varphi_0)\,\Phi(\pi-\varphi_0, \varphi_0)) \tag{7.2}$$

Formulas (7.1) and (7.2) in combination are called the optical theorem can be a check on the accuracy of the calculation.

8. One of the modifications of the procedure for calculating the field g by the formulas obtained above is the following: the boundary-value problem (2.2) and (2.3) is replaced by a system of mesh equations /7/

$$\psi(z_{i+1}) - (2 + (\lambda^2 - k^2(z_i))\,d^2)\,\psi(z_i) + \psi(z_{i-1}) = 0,\ i = 2, 3, \ldots, I-1 \tag{8.1}$$

where the mesh spacing $d = H/I$ is selected such that 10-15 nodes $z_i = id$ would arrive per period of eigenfunction oscillation. The first and last equations of the mesh system differ from (8.1) and take account of the boundary conditions (2.3). After getting rid of the radical $(\lambda^2 - k_1^2)^{1/2}$ in the determinant of this system, we obtain a polynomial whose roots are approximately identical with the eigenvalues of the "depth" problem lying on both sheets of the Riemann surface. A standard iteration procedure can be used to find these roots. The eigenfunctions that are solutions of the mesh system for already known λ_l are calculated from a trinomial recursion formula. To calculate the derivatives $\partial\psi_0'(H, \lambda_l)/\partial\lambda$ and $\partial\psi_0(H, \lambda_l)/\partial\lambda$, in (4.3), these recursion formulas should be differentiated, which will result in an inhomogeneous mesh system with the previous matrix.

The integrals I_{2p} are evaluated by means of (4.2). Since the series (4.2) converge slowly, a large number of terms must be summed. To accelerate the calculations starting with a certain number l^*, the asymptotic forms should be summed.

Comparison of the values of I_3 and I_5, evaluated by means of (4.1), with the known values of (4.4) is a check on the accuracy of the calculations. The optical theorem (7.1) and (7.2) is another checking identity.

If the spacing between the source (x_0, z_0), the inhomogeneities $(a_n, 0)$, and the point of observation (x, z) is large, then the integrals $I_{mnj}^{\pm}\,(m \neq n), J_{ms}, g_0$ can be replaced by sums of residues at the real poles $\lambda = \lambda_l$. Consequently, the field g is represented by a sum of normal waves propagating in a channel without damping

$$g(x, z) \approx \sum_{l=0}^{L} \alpha_l(x_0, z_0)\exp(i\lambda_l x)\,\psi_l(z) \tag{8.2}$$

The error of such a representation is due to neglecting the contributions of the complex poles $(\mathrm{Im}\,\lambda_l > 0)$ that decrease exponentially with distance, and the contribution of the side wave (the integral over the slit) that decreases as $O(|x|^{-3/2})$. If the distances between the plate defects are small, it is necessary to take account of the complex poles and integrals of the exponentially decreasing functions over the slit. We note that the inhomogeneities between which the spacings are much less than the wavelength in the half-space can still be replaced at the stage of formulating the problem of one point inhomogeneity.

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